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ANALYTICAL AND NUMERICAL APPROACH TO VIBRATION ANALYSIS OF LIQUID FILLED ALUMINUM THINCYLINDRICAL SHELLS

P. KIRAN KUMAR[1] & J. V. SUBRAHMANYAM[2]

[1]*Assistant Professor, Department of Mechanical Engineering, CBIT, Hyderabad, India*

[2]*Rtd.Professor, Department of Mechanical Engineering, Osmania College of Engineering, Hyderabad, India*

ABSTRACT

Vibration behavior of empty and variable fluid levels, contained in aluminum cylindrical shells of 2mm and 3mm subjected to horizontal accelerations are considered. Mathematical expressions, which show the motion of cylinder were developed and modified by use of small amplitude wave approximations, enabling equations for the various modes of vibrations and natural frequencies to be obtained [11]. The expression for frequency is formulated by the consideration of fluid in the cylindrical shell. The results were compared by considering modal analysis of aluminum shells, modeled and analyzed using ANSYS. Natural frequencies for different mode shapes are developed for both 2mm and 3mm aluminum shells of empty and with variable water column from the base of the shell. Damping ratios were calculated using half power method. Natural frequencies predicted from analytical method correlated with ANSYS, were found to be in close agreement.

KEYWORDS: Thin Shells, Natural Frequency & Damping Ratio

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Original Article

1. INTRODUCTION

In the recent times, the behavior of storage tanks that carry liquid has got special attention because of vulnerability effects of storage tanks due to earthquakes, as they caused liquid spill out due to structural failure, environmental effects and leakages, all these are potential losses to the nation. To reduce these effects due to winds and seismic excitations, storage tanks are proposed to have mass dampers. In this regard, number of works was carried by several researchers, who studied dynamic nature of storage tanks carrying liquids. Most of the works fluid mass is added to the structure of the tanks, the dynamic behavior characteristics are obtained with analytical approaches by considering simplified geometries and boundary conditions. Some of the earlier works are as follows.

Kubenko and Koval'chuk (2009)[3]observed in their analytical and experimental work that, shells with damping models subjected to axial and combined loads, there is big gap in the dynamic behavior, mainly because of geometric imperfections.

Mallon et al 2010[5] used Donnell's nonlinear shell theory in his analytical work, for the study of cylindrical shell with orthotropic material for multimode expansion to discretize PDE to ODE. In this theoretical work, shell and electromechanical shaker interaction was considered in the model. They also carried out experimental work. There is small gap between experimental and analytical study.

Pellicano (2005) [6] conducted experiments when shell of polymeric material with rigid mass on the topis subjected to the base excitation. Author claims that, rigid mass undergoes large amplitude while cylinder excited at its first mode of vibration with low base excitation about 10g.

Pellicano, 2007[8]in his further work, hefully considered problem in a linear Point of view. The nonlinear phenomena of shell were subjected to base excitation, with sine excitation to the electro mechanical shaker table. Shaking the shell due to sine excitation from the bottom induces a vertical motion of the top disk, which causes axial loads due to inertia forces. Due to these axial loads, the shell generally subjected to axial-symmetric deformations; but, in some conditions, it is noted that a violent resonant phenomenon takes place experimentally, by a strong energy transfer from low to high frequencies and large amplitude of vibration. Also, an interesting saturation phenomenon is noticed that the response of the top disk was completely flat, as the excitation frequency was changed around the shell base.

Vijayarachavan and Evan-Iwanowski, 1967[9] studied parametric instability analysis of circular cylindrical shell subjected to seismic excitation, by experimentally and analytically. They considered vertical circular cylinder when the base was axially excited, using an electro mechanical shaker. In this study, the in-plane inertia was variable along the shell axis, when the base is harmonically excited; it gives rise to a parametric excitation. Experimental and analytical instability regions were compared for first axisymmetric mode resonance. In experimental work, glass fiber-reinforced plastic cylindrical shell, both empty and water filled were subjected to the nonlinear dynamic deformation of the elastic wall, because of hitting by kinematic, two frequency load are studied.

The present work considers the general problem of thin circular cylindrical shell of 2mm and 3mm subjected to horizontal acceleration, containing different levels of in viscid fluid. In order to obtain equations for free surface displacements, Small amplitude wave approximations were considered. The fundamental natural frequencies of vibration of thin circular cylinder made of aluminum are calculated by determining the added mass of the fluid to the shell structure. Different modes of vibration of the cylindrical shell are obtained, with this, added mass of the liquid is added up with the structural mass. The natural frequencies were obtained analytically and using finite element approach, and are compared.

2. MATHEMATICAL FORMULATION

A. Mathematical Formulation for Determining the Fundamental Natural Frequencies of the Circular Cylindrical Shell

In the view of structural point, while considering the shell expressions, the axial component of motion is not considered, and so are the variations of displacement field, axially. Therefore, radial and tangential Love equations are written as [1]:

$$\frac{E_s \cdot e}{1-\nu^2}\left\{\frac{U}{R^2}+\frac{e^2}{12\cdot R^2}\left(\frac{\partial^4 U}{R^2\partial\theta^4}-\frac{\partial^3 V}{R^2\partial\theta^3}\right)+\frac{\partial V}{R^2\partial\theta}\right\}+\rho_s\cdot e\cdot\frac{\mathrm{d}^2 U}{\mathrm{d}t^2}=p(R,\theta;t) \tag{1}$$

$$\frac{E_s \cdot e}{1-\nu^2}\left\{\left(1+\frac{e^2}{12\,R^2}\right)\left(\frac{\partial^2 V}{R^2\partial\theta^2}\right)+\frac{\partial U}{R^2\partial\theta}-\frac{e^2}{12\,R^2}\cdot\frac{\partial^3 V}{R^2\partial\theta^3}\right\}-\rho_s\cdot e\cdot\frac{\mathrm{d}^2 V}{\mathrm{d}t^2}=0 \tag{2}$$

Assuming hoop strain to be negligible, Love eqn. in radial direction is reduced to:

$$\frac{Es\cdot e^3}{12\left(1-\nu^2\right)R^4}\left(\frac{\partial^4 U}{\partial\theta^4}+\frac{\partial^2 U}{\partial\theta^2}\right)+\rho_s\cdot e\cdot\frac{\mathrm{d}^2 U}{\mathrm{d}t^2}=p(R,\theta;t) \tag{3}$$

The mode shapes are of the following admissible type [1]:

$$\left\{u_n(\theta) = \alpha_n \cos(n\cdot\theta) + \beta_n \sin(n\cdot\theta); v_n(\theta) = \alpha_n \cos(n\cdot\theta) + b_n \sin(n\cdot\theta)\right\} \; n = 1, 2,\ldots\ldots \tag{4}$$

This can be conveniently split into two orthogonal families of mode shapes:

$$u_n{}^1(\theta) = \cos n\cdot\theta; v_n{}^1(\theta) = -\frac{1}{n}\sin n\theta; \; u_n{}^2(\theta) = \sin n\cdot\theta; v_n{}^2(\theta) = -\frac{1}{n}\cos n\theta; \tag{5}$$

The corresponding mass and stiffness coefficients per unit length are given by

$$m_s{}^{1,2}(n,n) = \rho s\cdot e\cdot\int_0^{2\pi}\left(1+\frac{1}{n^2}\right)\left\{\begin{matrix}\cos^2 n\theta\\ \sin^2 n\theta\end{matrix}\right\}Rd\theta = e\cdot\pi\cdot R\left(1+\frac{1}{n^2}\right) \tag{6}$$

$$k_s{}^{1,2}(n,n) = \frac{E_s\cdot e^3\cdot n^2\cdot(n^2-1)}{12(1-\nu^2)R^3}\cdot\int_0^{2\pi}\left(1+\frac{1}{n^2}\right)\left\{\begin{matrix}\cos^2 n\theta\\ \sin^2 n\theta\end{matrix}\right\}Rd\theta = \frac{E_s\cdot e^3\cdot\pi n^2\cdot(n^2-1)}{12(1-\nu^2)R^3} \tag{7}$$

The natural frequencies of the structure are given by [1]:

$$\omega_n = \sqrt{\frac{k_s(n,n)}{m_s(n,n)}} = \frac{n^2}{R}\left(\frac{e}{R}\right)\sqrt{\frac{Es}{12(1-\nu^2)}\frac{(n^2-1)}{(n^2+1)}} \tag{8}$$

Free and rigid in plane translations corresponds to n=1. The shapes constitute a subspace spanned by the two orthonormal vectors:

$$u_1 1 = \cos\theta; v_1 1 = -\sin\theta; \; u_2 1 = \sin\theta; v_1{}^2 = \cos\theta; \tag{9}$$

The mode shapes n>1 corresponds to axial bending

B. Considering the Effect of Fluid in the cylinder

To determine the added fluid mass coefficient inside the cylinder, a small strip model is used. It is assumed in the strip model, by considering a narrow strip between z and z+dz, located sufficiently away from the ends z=0 and z= H, we now turn our attention towards motion of the fluid forced by a radial vibration of the shell of the type.

$$U(\theta:t) = \sum_{n=1}^{\infty} q^1{}_n cosn\theta + q^2{}_n sinn\theta \tag{10}$$

Where, eq1 and eq2 stand for modal displacements of the shell Pressure is governed by the boundary value problem [1]:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p}{\partial\theta^2} = 0 \tag{11}$$

$$\frac{\partial p}{\partial r} = -\rho_f\Sigma\left(\ddot{q}^1{}_n cosn\theta + \ddot{q}^2{}_n sinn\theta\right); \; r = R \tag{12}$$

Splitting the above eqns. into sine and cosine families:

$$\frac{\partial^2 p^1{}_n}{\partial r^2} + \frac{1}{r}\frac{\partial p^1{}_n}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p^1{}_n}{\partial \theta^2} = 0 \tag{13}$$

$$\frac{\partial p^1{}_n}{\partial r} = -\rho_f \ddot{q}_n{}^1 cosn\theta;\ r = R$$

$$\frac{\partial^2 p^2{}_n}{\partial r^2} + \frac{1}{r}\frac{\partial p^2{}_n}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p^2{}_n}{\partial \theta^2} = 0 \tag{14}$$

$$\frac{\partial p^2{}_n}{\partial r} = -\rho_f \ddot{q}_n{}^2 cosn\theta;\ r = R \tag{15}$$

Solving the above equations, the pressure field can be determined as:

$$p^1{}_n(r, \theta, t) = -\frac{\rho_f H \ddot{q}_n{}^1}{nR^{n-1}} r^n cosn\theta \tag{16}$$

$$p^2{}_n(r, \theta, t) = -\frac{\rho_f H \ddot{q}_n{}^2}{nR^{n-1}} r^n sinn\theta \tag{17}$$

In the present problem, the added mass of the fluid to matrix, as obtained in the vacuum for structural mode and the mode shapes of the shell are the same. The common force exerted by the fluid on a shell strip with unit length is

$$Q_n = -\frac{\rho_f \ddot{q}_n{}^1}{n} HR^2 \int_0^{2\Pi} \vec{u}.\vec{u}.\cos^2 n\theta\, d\theta = -\frac{\rho_f . \Pi \cdot H \cdot R^2}{n} \ddot{q}_n{}^1 \tag{18}$$

Where $\vec{u}.$ is the unit normal vector in the radial direction, the added mass coefficients per unit length of the shell is given by:

$$M_a = \frac{\rho_f \pi \cdot HR^2}{n} \tag{19}$$

Where, the mode shape is normalized by the condition max $u_n{}^{(1,2)}$ (θ) =1 and $m_f = \rho\pi R^2 H$, stands for the physical mass of the fluid contained in the shell of unit length. The natural frequencies obtained through this analytical approach shown in results and discussion column.

3. ANSYS FORMULATION

In modeling section, cylinder is modeled as shell element both 2mm and 3 mm thickness. Finite element model is shown in figure-1 for pre processing model. Boundary condition for cylinders for all models fixed at ground position, and as well as cap modeled as shell element, as shown in figure-1. Inside, fluid modeled as fluid element and for fluid structure, interaction fluid velocity and density are given for modal analysis and random analysis. For the shell element for the Aluminum materials, Structural Solid with 3-D 8-Node - SOLID185 Element and fluid inside the shells as 3-D Acoustic Fluid -FLUID30 Element has been considered. Modeled shell with varying water levels (20% level variation from base to the shell height) as shown fig-1 has been meshed, using mesh attributes. The dimension of Aluminum hollow shell is of

2mm and 3mm thick, 100mm outer diameter, 300mm height is modeled with lid. The material properties of Aluminum with density (ρ) =2700kg/m^{3,} modulus of elasticity E =0.683x10^{11} N/m^2, Poisons ratio (ϑ) =0.34, element type SOLID185 Element is considered for shell element. Fluid with properties of Water density (ρ) =1000kg/m^{3,} sonic velocity=1498m/s^{2,} Viscosity= 0.00089 pa.s and element type for water as Fluid30, has been considered to prepare model, using ANSYS17.0

The physical problem has been converted into FEM problem using mapped mesh, and the shell element coupled with water element using coupled elements. The geometry is discretized into finite elements using solid 185 and fluid30 for structural and fluid, respectively. Outer peripheral nodes of fluid domain and inner peripheral nodes of structural container are normally coupled.

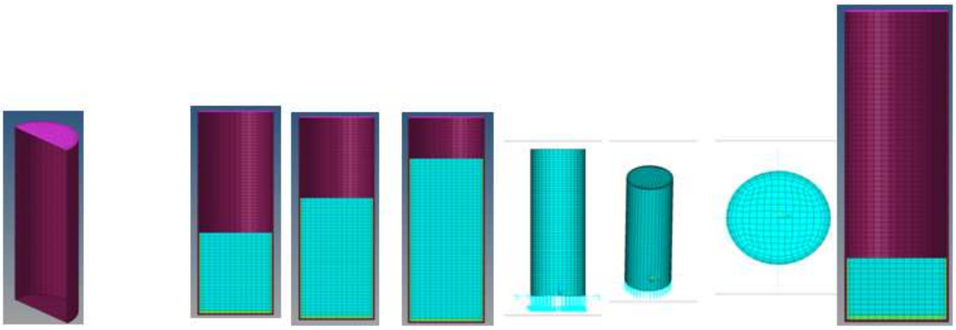
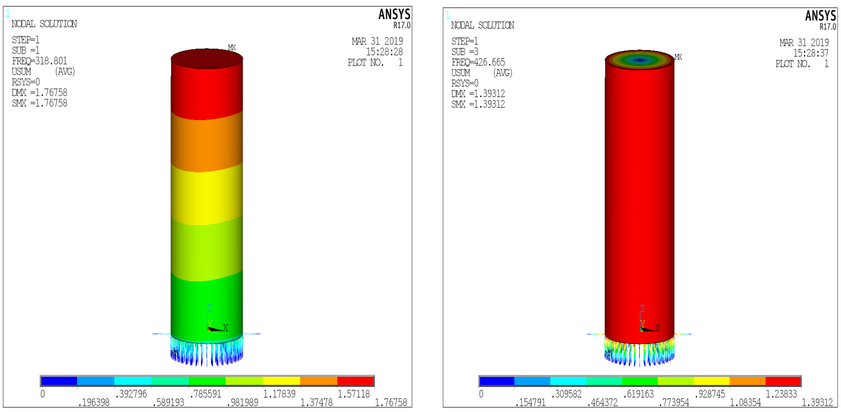


Figure 1: Shell Model with Variable Water Column in the shell, Shell with Boundary Condition Lid

4. RESULTS AND DISCUSSIONS

The mode shapes for above model, with results are shown in below figure-2&3 for empty and variable water column in the shell of 2mm thickness. Modal analysis performed for without water inside cylinder and with water column of60, 120,180,240 mm length, from bottom of cylinder. Extracted first four mode shapes and their natural frequencies in lanconze method are shown in below figure. Similarly, natural frequencies through modal analysis for 3mm aluminum shell were also developed, but mode shapes are not shown with figures but their natural frequencies with damping ratios are shown in table 1 &2 for first mode and second mode. It has been observed that acceleration ratio is reduced, with increase in thickness, the natural frequencies of the aluminum empty cylindrical shell increases, when water column in the cylindrical shell increases, the natural frequencies decrease because of mass of the water level added to the shell. And, damping ratios also increase with increases in water column.



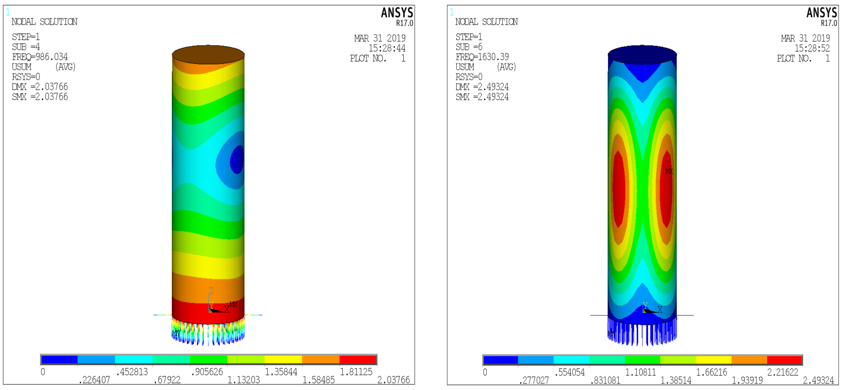
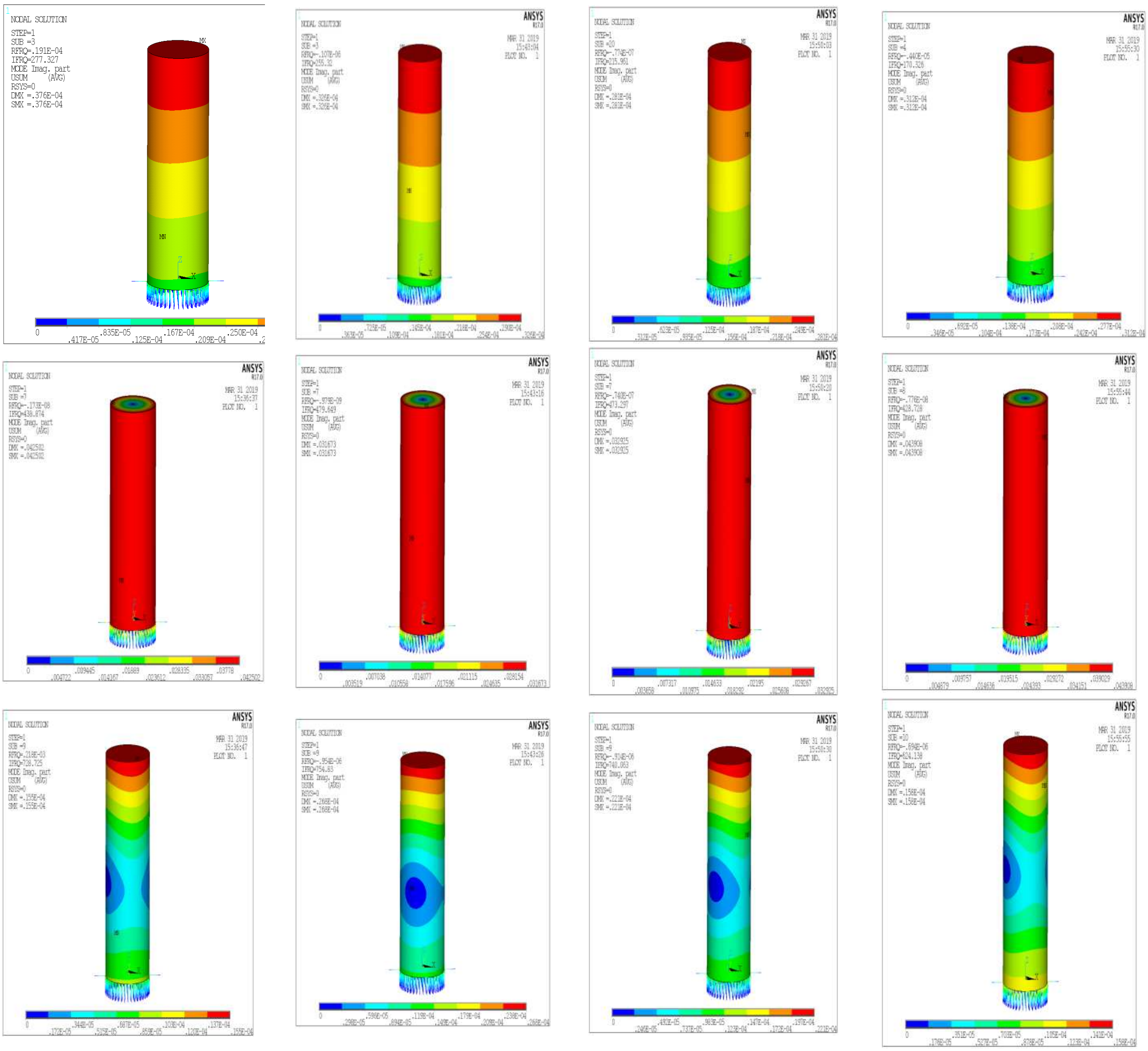


Figure 2: Four Mode Shapes of Empty Shell

a b c d



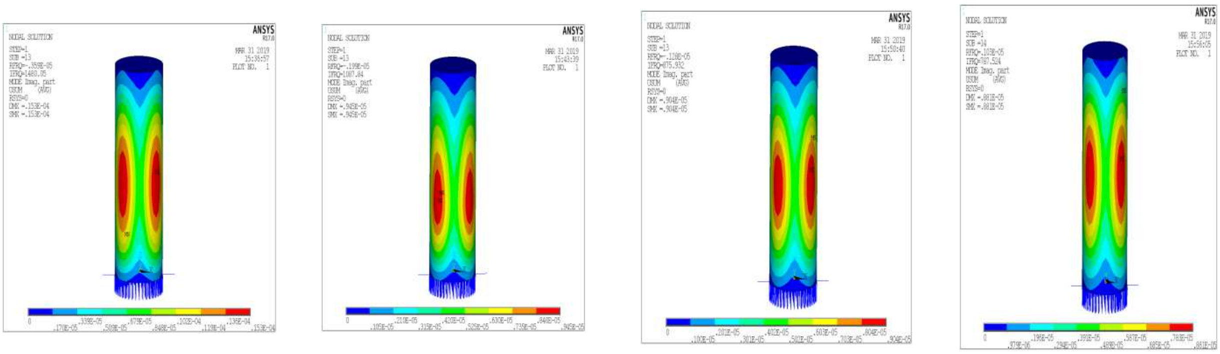


Figure 3: (a) 60 mm Water Column Shell Mode Shapes, (b) 120 mm Water Column Shell Mode Shapes, (c) 180 mm Water Column Shell Mode Shapes, (d) 240 mm Water Column Shell Mode Shapes

Table 1: Damping Ratios and Fundamental Natural Frequencies of Aluminum Circular Cylinders of 2mm and 3mm Thicknesses for Variable Water Column Heights

Thickness	Parameter	Water Column Length, Mm				
		0	60	120	180	240
2 mm	1st Natural Frequency Hz (Analytical)	315.4	276.2	250.3	221.8	178.6
	1st Natural Frequency Hz (FEM)	318.8	277.3	255.3	215.9	170.3
	% of error	1.07	0.4	1.99	2.66	4.64
	1st mode DampingRatio %	1.14	2.13	2.41	2.60	2.75
3 mm	1st Natural Frequency Hz (Analytical)	372.2	368.1	351.6	281.3	219.0
	1st Natural Frequency Hz (FEM)	378.4	375.6	348.3	279.4	224.0
	% of error	1.66	2.03	0.93	0.67	2.28
	1st mode DampingRatio %	2.94	2.63	2.81	2.73	2.54

Table 2: Damping Ratios and Second Natural Frequencies of Aluminum Circular Cylinders of Thicknesses for 2mm and 3mm Variable Water Column Heights

Thickness	Parameter	Water Column Length, Mm				
		0	60	120	180	240
2 mm	2nd Natural FrequencyHz(Analytical)	431.6	419.2	411.4	400.2	402.1
	2nd Natural FrequencyHz (FEM)	438.8	426.6	429.6	410.5	408.7
	% of error	1.66	1.76	4.42	2.57	1.64
	2nd mode DampingRatio %	1.50	3.15	3.20	3.38	3.45
3 mm	2nd Natural FrequencyHz(Analytical)	501.2	569.3	570.1	538.8	519.9
	2nd Natural FrequencyHz (FEM)	504.6	578.8	607.5	550.1	497.7
	% of error	0.67	1.66	6.56	2.09	4.27
	2nd mode Damping Ratio %	1.71	3.52	3.80	3.95	4.15

CONCLUSIONS

From table 1&2, it is clear that the natural frequencies of the aluminum empty cylinders increased as thickness is increased. The natural frequencies of the empty shell is decreased as the height of the water column increases, due to the added mass effect of the water column and presence of the water column in the hollow cylindrical shell. There is increase in damping ratios, as length of water column increased. The damping ratios are more for 3mm shell over 2mm thick shell. The natural frequencies of the cylinder were determined analytically, and compared with frequencies obtained using ANSYS, and are found to be in close agreement.

REFERENCES

1. *Arthur P. Boresi and Richard J. Schmidt, Advanced Mechanics of Materials 6th Edition, Wiley, 2002.*
2. *Francois Axisa and Jose Antunes – Modeling of Mechanical Systems Fluid Structure Interaction, Butterworth and Heinemann, Vol.3, 2007.*
3. *Kubenko, V.D., Koval'chuk, P.S., 2009. Experimental studies of the vibrations and dynamic stability of laminated composite shells. International Applied Mechanics 45 (5), 514–533.*
4. *Mallon, N.J., 2008. Dynamic Stability if Thin-Walled Structures: A Semi-Analytical and Experimental Approach. PhD Thesis, Eindhoven University of Technology Library, ISBN 978-90-386-1374-1.*
5. *Mallon, N.J., Fey, R.H.B., Nijmeijer, H., 2010. Dynamic stability of a base-excited thin orthotropic cylindrical shell with top mass: simulations and experiments. Journal of Sound and Vibration 329, 3149–3170.*
6. *Pellicano, F., 2005. Experimental analysis of seismically excited circular cylindrical shells. In: Proceedings of ENOC-2005, Fifth EUROMECH Nonlinear Dynamics Conference, Eindohven, the Netherlands, August 7–12.*
7. *Pellicano, F., Avramov, K.V., 2007. Linear and nonlinear dynamics of a circular cylindrical shell connected to a rigid disk. Communications in Nonlinear Science and Numerical Simulation 12 (4), 496–518.*
8. *F. PELLICANO 2007 Journal of Sound and Vibration303, 154–170. Vibrations of circular cylindrical shells: Theory and experiments.*
9. *Vijayarachavan, A., Evan-Iwanowski, R.M., 1967. Parametric instability of circular cylindrical shells. Journal of Applied Mechanics 34, 985–990.*
10. *Dinu James, Thanigaiarasu. G., 2011. Seismic Analysis on Cylindrical Tanks Subjected to Horizontal Acceleration. International Journal of Applied Research in Mechanical Engineering, Volume-1, Issue-1, 2011.*
11. *P.kirankumar, j.v.subrahmanyam., 2018, Analysis of base excited stainless steel cylindrical shell subjected to horizontal acceleration, IJMPERD special issue, june 2018, 14-21.*
12. *Erfan, O. S. A. M. A., El-Nasr, A., BA, A., & Al-mufadi, F. (2014). Erosion-corrosion behavior of AA 6066 aluminum alloy. IJME, 3, 15-24.*